Critical Loss Analysis with Differentiated Products

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Abstract: When products are differentiated, applying the standard critical loss formula to assess whether it is profitable for a hypothetical monopolist to impose a common price increase can lead to delineating an antitrust market that is too broad by setting a critical loss threshold that is too low. This error is particularly likely to occur when the products exhibit very different per unit profits, own price elasticities, and cross price elasticities. In particular, different per unit profits are a necessary condition for this error to occur and this difference is more likely to be driven by an asymmetry in prices than by an asymmetry in costs when own price elasticities are moderate in magnitude. In contrast, differences in the quantity sold of each product do not tend to lead to errors in market definition. Given the issues associated with the standard critical loss analysis, critical loss analysis with asymmetric price increases and the Gross Upward Price Pressure Index are practical alternative approaches for conducting market definition analysis when products in a candidate market are differentiated.

JEL classification codes: K21, L4

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I. Introduction

Critical loss analysis is an analytical tool commonly used in market definition analysis.[1] The U.S. Department of Justice and the Federal Trade Commission Horizontal Merger Guidelines ("Merger

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[1] For example, see Michael L. Katz and Carl Shapiro, "Critical Loss: Let's Tell the Whole Story," *Antitrust Magazine*, ABA Section of the Antitrust Law, Spring 2003, pages 49-56; James Langenfeld and Wenqing Li, "Critical Loss Analysis

Guidelines") discuss the use of critical loss analysis in the hypothetical monopolist test that defines a relevant market.[2] As explained in the Merger Guidelines, critical loss analysis can be used to implement the hypothetical monopolist test by evaluating whether "a hypothetical profit-maximizing firm, not subject to price regulation, that was the only present and future seller of those products ('hypothetical monopolist') likely would impose at least a small but significant and non-transitory increase in price ('SSNIP') on at least one product in the market, including at least one product sold by one of the merging firms."[3]

The standard, break-even critical loss is calculated using the following formula:[4]

$$\mathrm{CL} = \frac{t}{t+m} \tag{1}$$

where t is the percentage price increase, m is the price-cost margin, and CL is critical loss. The critical loss test is implemented by comparing the critical loss, CL, to the predicted percentage loss of sales, $\frac{\Delta Q}{Q}$, from a *t* percent price increase.[5] If the actual percentage of loss of sales is less than or equal to the critical loss, i.e., inequality (2) holds, it is profitable for the hypothetical monopolist to impose a uniform price increase on all products in the candidate market which implies the candidate market constitutes a relevant antitrust market.[6] On the other hand, if the actual percentage loss of sales is greater than the critical loss,

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in Evaluating Mergers," *The Antitrust Bulletin*, Summer 2001, pages 299-337; Daniel P. O'Brien and Abraham L. Wickelgreen, "A Critical Analysis of Critical Loss Analysis," *Antitrust Law Journal*, Volume 71, 2003, pages 161-184; Barry C. Harris, "Recent Observations About Critical Loss Analysis," updated June 25, 2015, available at https://www.justice.gov/atr/recent-observations-about-critical-loss-analysis.

[2] Horizontal Merger Guidelines, U.S. Department of Justice and the Federal Trade Commission, August 19, 2010, section 4.1.3.

[3] Horizontal Merger Guidelines, U.S. Department of Justice and the Federal Trade Commission, August 19, 2010, sections 4.1.1 and 4.1.3.

[4] Barry C. Harris and Joseph J. Simons, "Focusing Market Definition: How Much Substitution Is Necessary?" *Research in Law and Economics*, Volume 12, 1989, pages 207-226 at page 215.

[5] The Merger Guidelines call the actual loss of sales the predicted loss of sales given a *t* percent price increase. See Horizontal Merger Guidelines, U.S. Department of Justice and the Federal Trade Commission, August 19, 2010, section 4.1.3.

[6] Barry C. Harris and Joseph J. Simons, "Focusing Market Definition: How Much Substitution Is Necessary?" *Research in Law and Economics*, Volume 12, 1989, pages 207-226 at page 215. We often use the term "product market" in our discussions in this paper, but critical loss analysis can be applied to both product market definition and geographic market definition.

i.e., inequality (2) does not hold for many plausible values of t, the hypothetical monopolist cannot impose a profitable price increase on all products in a candidate market and the relevant antitrust market is broader than the candidate market.

$$\mathrm{CL} = \frac{t}{t+m} \geq \frac{\Delta Q}{Q} \tag{2}$$

This approach to defining a market deviates from the hypothetical monopolist test in the 2010 Merger Guidelines by requiring that a uniform or common price increase on all products is profitable rather than a price increase on "at least one product."[7] Thus, the standard critical loss formula will tend to result in a broader market, than a strict implementation of the hypothetical monopolist test that only requires a profitable price increase for one product sold by the merging firms.

Equations (1) and (2) used in the standard critical loss analysis also implicitly assume there is a single, undifferentiated product even though a candidate product market is often comprised of multiple differentiated products.[8] Consequently, standard critical loss analysis cannot be used to define a market without creating a representative product. A simple assumption people have used in mergers with multiple products is that there is a representative product whose profit margin can be calculated using the weighted average of the prices and costs of the actual products in the candidate market. For example, in FTC vs. Sysco and US Foods, one of the economic experts appeared to construct the critical loss using a weighted average of the merging parties' margins.[9] This assumption is innocuous if the products in a candidate market are undifferentiated with similar prices, costs, own price elasticities, and cross price elasticities, and one is interested in testing whether it would be profitable to increase the price of all

[7] Horizontal Merger Guidelines, U.S. Department of Justice and the Federal Trade Commission, August 19, 2010, sections 4.1.1.

[8] In the context of merger, a candidate market includes at least two products, one produced by each of the merging parties.

[9] FEDERAL TRADE COMMISSION, et. al., Plaintiffs, v. SYSCO CORPORATION, USF HOLDING CORP., and US FOODS, INC., Defendants. Civil Action No. 15-cv-00256 (APM) PUBLIC VERSION, PLAINTIFFS' CORRECTED PROPOSED FINDINGS OF FACT AND CONCLUSIONS OF LAW, at p. 66.

products by the same amount.[10] In this case, since all products have identical prices and costs, one can create a single representative product whose price and cost are equal to the price and cost of the individual products and apply the standard critical loss formula in (1) to implement a version of the hypothetical monopolist test using a common price increase.[11]

In principle, using a representative product to perform standard critical loss analysis in a differentiated product market could either raise or lower the critical loss threshold relative to an approach that allows for the products to have different prices, costs, and elasticities but imposes a common price increase. For example, if the representative product approach raises the critical loss threshold, it would tend to narrow the market because a given price increase is less likely to result in sufficient lost sales to make the price increase unprofitable, and potentially push it closer to a market defined using a strict implementation of the hypothetical monopolist test that only requires a profitable price increase for one product sold by the merging firms. However, as we show in the rest of this paper, the representative product approach actually tends to lower the critical loss threshold. Consequently, using a representative product will tend increase the size of the antitrust market above and beyond an approach that imposes a common price increase.

A necessary condition for the standard critical loss analysis under the representative product approach to incorrectly define an antitrust market is that the per unit profits of the products in the candidate market differ from each other. When this necessary condition holds and the products have different own price elasticities and/or cross price elasticities, the standard critical loss analysis will shift

[10] For the observation that the standard critical loss formula assumes a symmetric price increase, see Øystein Daljord, Lars Sørgard & Øyvind Thomassen, "The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro," *Journal of Competition Law & Economics*, Vol. 4, 2008, pages 263-270.
[11] See Appendix for details of the proof. Please note the own price elasticity for this single aggregated product is smaller than the own price elasticity for each individual product. See Daniel P. O'Brien and Abraham L. Wickelgreen, "A Critical Analysis of Critical Loss Analysis," *Antitrust Law Journal*, Volume 71, 2003, pages 161-184 at page 169.

more weight to the product whose reduction in profit due to a price increase is larger. As a result, the standard critical loss analysis tends to indicate a price increase is not profitable when in fact it is profitable to impose the price increase. On the other hand, when this necessary condition does not hold, i.e., when the per unit profits of the products in the candidate are the same, the standard critical loss analysis will not generate incorrect predictions about the profitability of a price increase even if the products in the candidate market have different own and cross price elasticities. Our analysis further shows that differences in quantities of each product do not lead to errors in antitrust market definition using the standard critical loss analysis. Moreover, the differences in per unit profits that can result in erroneous prediction using the standard critical loss is more likely to be driven by asymmetry in prices than asymmetry in costs when own price elasticities are moderate in magnitude.

To avoid this pitfall of applying the standard critical loss formula, we recommend conducting critical loss analysis using formulas that allow for asymmetric price increases and different margins for the products in the candidate market or by calculating the Gross Upward Price Pressure Index. These existing approaches to market definition analysis are both consistent with the Merger Guidelines in that they allow for increasing the price of one product rather than a common price increase across all products, and they do not require constructing a single representative product with a single margin,[12] which has often been used in antitrust matters to define markets.

II. Analysis of Profitability of a Common Price Increase Under Differentiated Products

Suppose there are two products in the candidate market and we are interested in asking the question whether it would be profitable for a monopolist to impose a common price increase across both products.

[12] See, e.g., James Langenfeld and Wenqing Li, "Critical Loss Analysis in Evaluating Mergers," *The Antitrust Bulletin*, Summer 2001, pages 299-337, and Øystein Daljord, Lars Sørgard & Øyvind Thomassen, "The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro," Journal of Competition Law & Economics, Vol. 4, 2008, pages 263-270.

This question is essentially what a standard, break-even critical loss analysis attempts to answer. Let us define:

p_1: price of product 1;

p_2: price of product 2;

C_1: constant marginal cost of product 1;[13]

C_2: constant marginal cost of product 2;

q_1: quantity of product 1;

q_2: quantity of product 2;

m_1: price-cost margin of product 1;

m_2: price-cost margin of product 2;

ε_1: absolute value of own price elasticity of product 1;

ε_2: absolute value of own price elasticity of product 2;

ε_{12}: cross elasticity of demand of product 1 with respect to the price of product 2;

ε_{21}: cross elasticity of demand of product 2 with respect to the price of product 1;

t: the percentage price increase across both products;

Δq_1: change in the quantity of product 1 after taking into account the cross price elasticity effect;

[13] When Harris and Simons developed the critical loss formula, the cost used in calculating the price-cost margin is average variable cost. However, they assumed the average variable cost was constant so the average variable cost is the same as the constant marginal cost. See Barry C. Harris and Joseph J. Simons, "Focusing Market Definition: How Much Substitution Is Necessary?" *Research in Law and Economics*, Volume 12, 1989, pages 207-226 at pages 214-215.

Δq_2: change in the quantity of product 2 after taking into the cross price elasticity effect.

The price increase of *t* percent is profitable if:

$$p_1(1+t)(q_1-\Delta q_1)-C_1(q_1-\Delta q_1)+p_2(1+t)(q_2-\Delta q_2)-C_2(q_2-\Delta q_2)$$

$$\geq p_1q_1-C_1q_1+p_2q_2-C_2q_2 \tag{3}$$

Inequality (3) holds if and only if:[14]

$$tp_1q_1+tp_2q_2 \geq [p_1(1+t)-C_1]\Delta q_1+[p_2(1+t)-C_2]\Delta q_2 \tag{4}$$

The left side of inequality (4) represents the increase in revenue as a result of the price increase while the right side of inequality (4) represents the decrease in the hypothetical monopolist's profit at the elevated price that can be attributed to the lost sales for product 1 and product 2. Therefore, inequality (4) shows that if the revenue increase from the price increase more than offsets the reduction in profits due to loss of sales for product 1 and product 2, then the price increase is profitable.

Assume prices of product 1 and product 2 are set to maximize the profits of product 1 and product 2 respectively before the price increase, using the Lerner Index from the first order conditions, we have:[15]

$$p_1=\frac{C_1\varepsilon_1}{\varepsilon_1-1} \tag{5}$$

$$p_2=\frac{C_2\varepsilon_2}{\varepsilon_2-1} \tag{6}$$

[14] See Appendix for details. This inequality can be re-arranged as $tp_1(q_1-\Delta q_1)+tp_2(q_2-\Delta q_2) \geq [p_1-C_1]\Delta q_1+[p_2-C_2]\Delta q_2$ so that the increase in revenue as a result of the price increase on the left side is evaluated at unit sales after sales have declined as price rises and the decrease in the hypothetical monopolist's profit due to lost sales on the right side is evaluated at the price level before the price increase is imposed. However, as we show later, the expression in inequality (4) facilitates the comparison to the standard critical loss analysis so that one can easily assess when the standard critical loss analysis will diverge from the differentiated products critical loss analysis.

[15] See Appendix for details.

Given the own price elasticities and cross price elasticities, the changes in the quantity of products 1 and 2 can be expressed as:[16]

$$\Delta q_1 = q_1 t\varepsilon_1 - q_1 t\varepsilon_{12} \tag{7}$$

$$\Delta q_2 = q_2 t\varepsilon_2 - q_2 t\varepsilon_{21} \tag{8}$$

Substitute (5)-(8) into (4) and re-arrange the terms, (4) holds if and only if:[17]

$$t\left(\frac{C_1\varepsilon_1}{\varepsilon_1-1}q_1 + \frac{C_2\varepsilon_2}{\varepsilon_2-1}q_2\right) \geq \frac{t\varepsilon_1+1}{\varepsilon_1-1}[C_1(q_1 t\varepsilon_1 - q_1 t\varepsilon_{12})] + \frac{t\varepsilon_2+1}{\varepsilon_2-1}[C_2(q_2 t\varepsilon_2 - q_2 t\varepsilon_{21})] \tag{9}$$

Inequality (9) is the correct inequality for to assessing if a hypothetical monopolist can impose a common price increase of *t* percent under product asymmetry. If inequality (9) holds, it is profitable for the hypothetical monopolist to impose a price increase of *t* percent and the candidate market is a relevant antitrust market. Otherwise, the price increase is not profitable and the relevant antitrust market may be broader than the candidate market although it is still possible it is profitable to raise the price of a single product as we show below, which would pass the hypothetical monopolist test under the Merger Guidelines. We call inequality (9) the analysis of profitability of a common price increase under differentiated products or "differentiated products critical loss analysis".

III. The Standard Critical Loss Analysis with the Differentiated Products Under the Representative Product Approach

Instead of the differentiated products critical loss analysis, one common practice is to aggregate the asymmetric products to a single representative product. Under this representative product approach, one calculates the price-cost margin using the weighted average price and weighted average cost, and then

[16] We assume product 1 and product 2 are substitutes so the cross price elasticities are positive. These elasticities are point elasticities.
[17] See Appendix for details.

applies the standard critical loss analysis in inequality (2) to assess if a *t* percent common price increase is profitable. Applying the standard critical loss analysis to a representative product, we will have:

$$\mathrm{CL} = \frac{t}{t+\bar{m}} \geq \frac{\Delta Q}{Q} = \frac{\Delta q_1 + \Delta q_2}{q_1 + q_2} \tag{10}$$

where $\bar{m}$ is the average price-cost margin of the single aggregated product. Define the weighted average price and weighted average cost of products 1 and 2 as:

$$\bar{p} = \frac{p_1 q_1 + p_2 q_2}{q_1 + q_2} \tag{11}$$

$$\bar{C} = \frac{C_1 q_1 + C_2 q_2}{q_1 + q_2} \tag{12}$$

Then the average price-cost margin of the single aggregated product can be calculated as:

$$\bar{m} = \frac{\bar{p} - \bar{C}}{\bar{p}} \tag{13}$$

Substitute (5)-(6) into (11)-(12) and then substitute (11)-(12) into (13) and re-arranged the terms, the average price-cost margin of the single aggregated product can be expressed as:[18]

$$\bar{m} = \frac{C_1 q_1 \frac{1}{\varepsilon_1 - 1} + C_2 q_2 \frac{1}{\varepsilon_2 - 1}}{C_1 q_1 \frac{\varepsilon_1}{\varepsilon_1 - 1} + C_2 q_2 \frac{\varepsilon_2}{\varepsilon_2 - 1}} \tag{14}$$

Substitute (7)-(8) and (14) into inequality (2) in the standard critical loss analysis, and we find the inequality assessing the profitability of a common price increase under product asymmetry becomes:

$$\mathrm{CL} = \frac{t}{t + \frac{C_1 q_1 \frac{1}{\varepsilon_1 - 1} + C_2 q_2 \frac{1}{\varepsilon_2 - 1}}{C_1 q_1 \frac{\varepsilon_1}{\varepsilon_1 - 1} + C_2 q_2 \frac{\varepsilon_2}{\varepsilon_2 - 1}}} \geq \frac{\Delta Q}{Q} = \frac{q_1 t \varepsilon_1 - q_1 t \varepsilon_{12} + q_2 t \varepsilon_2 - q_2 t \varepsilon_{21}}{q_1 + q_2} \tag{15}$$

Comparing (9) and (15), it is obvious that (9) does not necessarily imply (15) and (15) does not necessarily imply (9) either. This result indicates that if one simply uses the standard critical loss formula with

[18] See Appendix for details.

weighted average prices and costs, it is possible that one will set the critical loss threshold either lower or higher than the threshold derived from a common price increase with differentiated products.

IV. The Divergence of the Standard Critical Loss Analysis from the Differentiated Products Critical Loss Analysis

To see when standard critical loss will differ from differentiated products critical loss, we start with the conditions that must be met for a price increase in differentiated products critical loss and standard critical loss analysis to be profitable. For differentiated product critical loss, a price increase will only be profitable if inequality (4) holds, namely,

$$tp_1q_1 + tp_2q_2 \geq [p_1(1+t) - C_1]\Delta q_1 + [p_2(1+t) - C_2]\Delta q_2.$$

The equivalent condition for standard critical loss is shown in equation 16 below[19].

$$t\frac{p_1q_1+p_2q_2}{q_1+q_2}(q_1+q_2) \geq \frac{p_1q_1+p_2q_2}{q_1+q_2}(1+t)(\Delta q_1+\Delta q_2) - \frac{C_1q_1+C_2q_2}{q_1+q_2}(\Delta q_1+\Delta q_2) \quad (16)$$

After some algebra, this equation simplifies to [20]

$$tp_1q_1 + tp_2q_2 \geq [p_1(1+t) - C_1]\frac{q_1}{q_1+q_2}(\Delta q_1+\Delta q_2) + [p_2(1+t) - C_2]\frac{q_2}{q_1+q_2}(\Delta q_1+\Delta q_2) \quad (17)$$

Notice that the left sides of inequalities (4) and (17), which represents the rise in revenue as a result of the price increase, are identical, and the right side of inequality (17), which represents the reduction in profits due to lost sales, is similar to that in (4), but has different weights on each of the profit terms. From this we can infer that in order for differentiated products critical loss to differ from standard critical loss, it must be the case that the two products have different per unit profits. This result implies that a necessary condition for standard critical loss to deviate from differentiated products critical loss is that

$$p_1(1+t) - C_1 \neq p_2(1+t) - C_2 \quad (18)$$

[19] See Appendix for details.
[20] See Appendix for details.

This result is practically useful because it means that if a practitioner knows the per unit profits of the products are very similar, then one can apply standard critical loss to a representative product as a screening tool even though the products are differentiated, knowing that the market definition threshold set by standard critical loss will not differ significantly from differentiated products critical loss.

Comparing inequalities (4) and (17), we can also see that the standard critical loss and differentiated products critical loss will put different weight on the profit terms whenever

$$\frac{q_1}{q_1+q_2}(\Delta q_1 + \Delta q_2) \neq \Delta q_1 \text{ and } \frac{q_2}{q_1+q_2}(\Delta q_1 + \Delta q_2) \neq \Delta q_2 \tag{19}$$

Without loss of generality, focusing on the equation for the weight on the per unit profit of product 1, we can simplify this expression to[21]

$$\frac{q_1 \Delta q_2 - \Delta q_1 q_2}{q_1+q_2} \neq 0 \tag{20}$$

Using equations (7) and (8) and some algebra, we can re-write this as[22]

$$\frac{q_1 q_2 t[(\varepsilon_2 - \varepsilon_1) - (\varepsilon_{21} - \varepsilon_{12})]}{q_1+q_2} \neq 0 \tag{21}$$

Equation (21) implies the weights standard critical loss places on each product will be the same as those in differentiated products critical loss when the products are symmetric in their own price elasticities and cross price elasticities, i.e., $\varepsilon_2 = \varepsilon_1$ and $\varepsilon_{21} = \varepsilon_{12}$. If the elasticities differ, then differentiated product critical loss will differ from standard critical loss except in the knife-edge case where $\varepsilon_2 - \varepsilon_1 = \varepsilon_{21} - \varepsilon_{12}$. Importantly, Equation (21) also shows that differences in quantities do not drive the difference in the standard critical loss and differentiated products critical loss because the differences in the weights are determined by the differences in own and cross price elasticities only.

[21] See Appendix for details.
[22] See Appendix for details.

The analyses above provide guidance for practitioners on when standard critical loss will be a good approximation of differentiated products critical loss. As long as the own price elasticities and cross price elasticities of the products are very similar, standard critical loss under a representative product approach will approximate differentiated products critical loss well even if per unit profits of the two products are different.

On the other hand, if the products have different per unit profit as well as different own price elasticities and/or different cross price elasticities, standard critical loss analysis under a representative product approach is likely to lead to different conclusions on market definition from differentiated products critical loss analysis. Moreover, as we discuss below, standard critical loss analysis under a representative product approach generally shifts more weight to the product whose reduction in per unit profit due to a price increase is larger. This implies that when per unit profits are significantly different, a small shift in weight due to small differences in own price elasticities or cross price elasticities between product 1 and product 2 can create divergence between the two methods.

V. Markets Defined Using Standard Critical Loss Will Tend to Define a Narrower Market than Differentiated Products Critical Loss

Now that we know the circumstances under which standard critical loss will differ from differentiated products critical loss, we show whether standard critical loss will tend to define broader or narrower markets than differentiated products critical loss. From equation (19) and (21), it is clear that standard critical loss will put more weight on the per unit profit of product 1 than differentiated products critical loss does when

$$\frac{q_1 q_2 t[(\varepsilon_2-\varepsilon_1)-(\varepsilon_{21}-\varepsilon_{12})]}{q_1+q_2} > 0 \qquad (22)$$

And less weight on the profit of product 1 when

$$\frac{q_1 q_2 t[(\varepsilon_2-\varepsilon_1)-(\varepsilon_{21}-\varepsilon_{12})]}{q_1+q_2} < 0 \tag{23}$$

Recall the own price elasticity was defined using the Lerner index, which means the higher the firm's per-unit profit, the lower the absolute value of own price elasticity in general. For simplicity assume $\varepsilon_{21} = \varepsilon_{12}$ and suppose that the per unit profit of product 2 is smaller than product 1. Then through Lerner index, we generally expect that $\varepsilon_2 > \varepsilon_1$.[23]

Therefore, when the per unit profit of product 2 is smaller than product 1, standard critical loss puts less weight on product 2, but more weight on product 1 than differentiated products critical loss. This means that standard critical loss puts more weight on the product with the higher per unit profit and less weight on the product with the lower per unit profit than differentiated products critical loss. In this context, the higher the per unit profit, the greater the reduction in profits from a price increase for a given change in quantity. As a result, standard critical loss generally shifts more weight to the product whose reduction in profits due to a price increase is larger for the same change in quantity, which means the standard critical loss can indicate the price increase is not profitable even the price increase is actually profitable based on the differentiated products critical loss. In other words, standard critical loss tends to define a narrower market than the differentiated products critical loss.

VI. Comparing the Standard Critical Loss Analysis and the Differentiated Products Critical Loss Analysis using Simulations

To assess our analytical results regarding when standard critical loss analysis and the differentiated products critical loss analysis diverge under product asymmetry, we conduct a Monte Carlo style simulation with 50,000 draws. In the simulation, we assume that the absolute values of the own price

[23] This relationship will not always hold because it is possible that a product with a higher per unit profit has a lower profit margin. But this relationship will hold if the costs per unit for product 1 and product 2 are the same. See Appendix for details.

elasticity is greater than 1, price is greater than cost, and each firm's own price elasticity is evaluated at the pre-merger profit-maximizing price. We draw quantities uniformly from 100-250, costs uniformly from 5-10, own price elasticities uniformly from 1.3-2.5.[24] The margins are computed from the elasticities using the Lerner index, prices are computed from the costs and margins, and the cross elasticities are assumed to range from 0.0001 to a quarter of the absolute value of own price elasticity. We use a 5 percent price increase to compute the critical loss. Given these parameter inputs, the prices of product 1 and product 2 are calculated using (5) and (6) above and the changes in the quantities of product 1 and product 2 are calculated using (7) and (8) above.

Table 1 shows that the standard critical loss analysis and the differentiated products critical loss analysis produce consistent results a majority of the time for the values we simulate. However, about 19% of the time, the standard critical loss analysis diverges from the differentiated products critical loss analysis. When the standard critical loss analysis and the differentiated products critical loss analysis diverge, the standard critical loss analysis almost always indicates the price increase is not profitable even though it is actually profitable for the hypothetical monopolist to impose the price increase based on the differentiated products critical loss analysis. In other words, the standard critical loss analysis using a representative product can incorrectly lead to the conclusion the candidate market does not constitute a relevant antitrust market when it in fact does. It tends to exacerbate the problem inherent in analyses that require a common price increase by making the market even broader than implied by a common price increase critical loss test. There are a few instances in the simulation results where the price increase is profitable according to the standard critical loss analysis and not profitable according to the differentiated products critical loss analysis, but they only account for 0.69% of the observations. These

[24] Some researchers have stated that price-cost margins are often in the range of 40 to 70 percent, which imply an absolute value of own price elasticity of 2.5 to 1.4 based on the Lerner Index. Gregory J. Werden, for example, has stated that "price-cost margins in real-world antitrust matters commonly are in the 40 to 70 percent range." Gregory J. Werden, "Demand Elasticities in Antitrust Analysis," *Antitrust Law Journal*, Volume 66, 1998, pages 363-414 at page 390.

simulation results confirm our analytical results that indicate that a standard critical loss analysis will tend to define a market that is too broad when it diverges from the differentiated products critical loss analysis.

Next, we investigate under what conditions, the standard critical loss analysis is likely to diverge from the differentiated products critical loss analysis.[25] Figure 1 shows binned scatter plots of the relationship between firm 1 and 2's per unit profits, prices, quantities, costs, own price elasticities and cross price elasticities in the simulations for situations when the standard critical loss produces the correct results, i.e. the same result as the differentiated products critical loss analysis, and circumstances when it does not.[26] If our analytical results are correct, the divergence between standard critical loss and differentiated products critical loss would be driven by asymmetries in the products' per unit profits and elasticities, but not by asymmetries in the products' quantities.

The results in Figure 1 are broadly consistent with the analytical results. First, they show that for the cases where standard critical loss correctly predicts whether or not a price increase will be profitable, there is no asymmetry between the prices, quantities, costs, per unit profits, or elasticities of the two products. This result is expected because when the predictions are correct, there should not be an asymmetry in the product characteristics, so we should not expect to see, e.g., that the own price elasticity of product 1 is typically high when the own price elasticity of product 2 is low.

As can be seen in Figure 1, however, when standard critical loss incorrectly predicts whether or not a price increase will be profitable, we see clear evidence of an asymmetry for prices, per unit profits, own price elasticities, and cross price elasticities. For example, when standard critical loss incorrectly predicts

[25] In this investigation, we will only focus on the instances where the standard critical loss analysis indicates the price increase is not profitable while the differentiated products critical loss analysis shows that the price increase is profitable.

[26] Binned scatterplots are created by "group[ing] the x-axis variable into equal-sized bins, comput[ing] the mean of the x-axis and y-axis variables within each bin, then creat[ing] a scatterplot of these data points." They are used to show the relationship between variables when data contain a large number of observations that could otherwise obscure how the x-variable and y-variable are related. See https://michaelstepner.com/binscatter/.

whether a price increase will be profitable and the per unit profit of product 1 is high, the per unit profit of product 2 tends to be low. The asymmetry in prices, per unit profits, own price elasticities, and cross price elasticities when standard critical loss diverges differentiated products critical loss is consistent with our analytical results. The negative relationship between the cross price elasticities is not quite as sharp as that between the own price elasticities. One reasons for this may be that we constrain the cross price elasticities to be smaller than the own price elasticities, so the own price elasticities will tend to have bigger effect on whether or not standard critical loss analysis is correct than the cross price elasticities.

In theory, both the asymmetry in prices and the asymmetry in costs can lead to the difference in per unit profits. Therefore, it is puzzling that we find a price asymmetry but not a cost asymmetry when standard critical loss incorrectly predicts whether or not a price increase will be profitable. However, as we show below, given moderate own price elasticities, asymmetries in prices are more likely than asymmetries in costs to lead to differences in per unit profits that are large enough to cause errors in market definition using standard critical loss. To see this implication, we examine the relationship between the difference in prices and own price elasticities and the relationship between the difference in costs and own price elasticities.

Given the Lerner Index relationship between price, cost, and own price elasticity, holding cost constant, the difference in prices can be expressed as[27]

$$p_1 - p_2 = C\left[\frac{\varepsilon_2 - \varepsilon_1}{(\varepsilon_1 - 1)(\varepsilon_2 - 1)}\right] \tag{24}$$

Similarly, holding price constant, the difference in costs can be expressed as[28]

$$C_1 - C_2 = p\left[\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 \varepsilon_2}\right] \tag{25}$$

[27] See Appendix for details.
[28] See Appendix for details.

Since $\varepsilon_1 > 1$ and $\varepsilon_2 > 1$, we have $(\varepsilon_1 - 1) < \varepsilon_1$ and $(\varepsilon_2 - 1) < \varepsilon_2$. Given the moderate own price elasticities in the simulation, $(\varepsilon_1 - 1)(\varepsilon_2 - 1)$ can be significantly smaller than $\varepsilon_1\varepsilon_2$. Therefore, the absolute value of the second term in the bracket in (24) can be significantly larger than the absolute value of the second term in the bracket in (25).

Now, suppose we have a random draw 1 that produced the same costs for product 1 and product 2, but a lower own price elasticity for product 1 than for product 2.[29] Equation (24) implies that the difference in prices, hence, the difference in per unit profits, can be very large, which is more likely to result in the divergence between standard critical loss and differentiated products critical loss. Next, assume we have a random draw 2 that produced the same set of own price elasticities as those in random draw 1, but different costs such that the prices for product 1 and product 2 are the same.[30] Equation (25) implies that the difference in cost, hence, the difference in per unit profits, tends to be smaller in this case, which makes it less likely standard critical loss diverges from differentiated products critical loss. Therefore, asymmetries in prices are more likely to lead to incorrect market definition than asymmetries in costs given the moderate own price elasticity range in the simulation.

However, as own price elasticities become larger, both $\frac{1}{(\varepsilon_1-1)(\varepsilon_2-1)}$ and $\frac{1}{\varepsilon_1\varepsilon_2}$ will be significantly less than 1. Consequently, both the asymmetry in prices and the asymmetry in costs may cause differences in per unit profits that are large enough to lead to incorrect market definition using standard critical loss.

We check whether this analysis is correct in Figure 2 which replicates the analysis in Figure 1 except it uses an own price elasticity range of 3 to 5 instead 1.3 to 2. The results in Figure 2 are similar to those in Figure 1 except for the cost plot which shows asymmetry in costs when standard critical loss incorrectly

[29] The costs do not have to be exactly the same. As long as the costs are similar, the implications discussed above will apply.

[30] The prices do not have to be exactly the same. As long as the prices are similar, the implications discussed above will apply.

predicts whether a price increase will be profitable. This result confirms that as own price elasticities become larger, both asymmetry in prices and asymmetry in costs can result in large enough differences in per unit profits to lead to standard critical loss incorrectly defining a market.[31]

Taken together, the simulation confirms our analytical results that asymmetries in per unit profits, own price elasticities and cross price elasticities are the key drivers that makes the standard critical loss analysis diverge from the differentiated products critical loss analysis.

To test of the robustness of the simulation results, we conduct additional simulations using multiple different ranges of own price elasticities. Table 2 compares the likelihood of divergence between the standard critical loss analysis and the differentiated products critical loss analysis with different own price elasticity ranges. The patterns of divergence between these two critical loss analyses remain the same as the range of own elasticity varies: when the standard critical loss analysis and the differentiated products critical loss analysis diverge, the standard critical loss analysis almost always indicates the price increase is not profitable even though it is actually profitable for the hypothetical monopolist to impose the price increase based on the differentiated products critical loss analysis. Moreover, Table 2 indicates that the lower the range of the own price elasticity, the more likely that the standard critical loss will diverge from differentiated products critical loss analysis, which is consistent with our analytical results. Lower own price elasticity generally implies higher per unit profit because of the Lerner Index relationship between own price elasticity and profit margin. Since the standard critical loss tends to put more weight on the product whose reduction in per unit profit due to lost sales is larger, as its per unit profit rises, even small asymmetries in own and cross price elasticities can result in changes in weights that will make the standard critical loss analysis diverge from the differentiated products critical loss analysis.

[31] If the own price elasticity ranges from 2 to 4, the lack of cost asymmetry when standard critical loss leads to incorrect market definition persists. The results are available from the authors upon request.

VII. The Divergence of the Standard Critical Loss Analysis from the Differentiated Products Critical Loss Analysis Is Likely to Occur in Practice

Our analysis shows that when products in a candidate market are differentiated, especially when the products have different demand elasticities, aggregating the products to a single representative product can incorrectly define the relevant antitrust markets. This issue can arise when defining markets in practice because products that are close substitutes, and hence are likely to be included together in a candidate market, can have differences in own price elasticities whose magnitudes are similar to those that can result in inconsistency between the standard critical loss analysis and the differentiated products critical loss analysis.

For example, differences in own price elasticities between different powdered detergent products can be as high as 2.6 according to a research by Song and Chintagunta.[32] In a study of demand elasticities for bathroom tissue products, Langenfeld, Li, and Yang found that the differences between the absolute values of own price elasticities between two branded bathroom tissue products can be as high as 1.5.[33] Both of these differences in own price elasticities are larger than the maximum own price elasticity difference of 1.2 used in the simulation. A similar issue can arise when critical loss is computed for cluster markets where one groups products that are not substitutes into one antitrust market, e.g., various services offered by a bank into a commercial banking product market, and price elasticities may differ across the various products.[34]

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[32] Inseong Song and Pradeep K. Chintaguanta, "Measuring Cross-Category Price Effects with Aggregate Store Data," *Management Science*, Vol. 52, No. 10, October 2006, pages 1594–1609 at pages 1600 and 1605.
[33] James Langenfeld, Wenqing Li, and Sophie Yang, Chapter 2, "Bayer or Walgreen? The Relationship of Premium and Value Brands in the United States," in *Brands, Competition Law and IP*, edited by Deven R. Desai, Ioannis Lianos, and Spencer Weber Waller, Cambridge University Press, 2015, Table 2.4 at page 44.
[34] Ian Ayers, "Rationalizing Antitrust Cluster Markets," *Yale Law Review,* Volume 95, No. 1, 1985, pages 109-125.

For practical reasons, aggregating certain products together may be necessary in the evaluation of relevant product markets. Even though strictly speaking, each item that has its own price can be treated as a separate product from other items, treating each item as an individual product can make the implementation of the hypothetical monopolist test intractable because there can be many dozens or even hundreds of items in a narrow category. For example, in the over-the-counter analgesics category, there are a few major brands such as Tylenol, Advil, Aleve, and Motrin. Within each brand, there are different formulations such as tablets, capsules, liquid gels, etc. Within each formulation, a given brand can be sold in bottles with different counts. As a result, the number of products can multiply quickly and product aggregation may be necessary. Our analysis suggests that one should exercise caution when aggregating products together when assessing the relevant product market, especially when the per unit profits, own price elasticities, and cross price elasticities of the products in a candidate market can differ from each other noticeably.

VIII. Alternative Market Definition Analyses When Products Are Differentiated

Given the potential issue in applying standard critical loss formula when products are differentiated and the complexity of the differentiated products critical loss analysis of profitability of a common price increase, one should consider other tools for market definition analysis when products are differentiated. One alternative is to perform critical loss analysis with asymmetric price increases using results from Langenfeld and Li (2001), which are consistent with the Merger Guidelines in that they allow one to measure the critical loss from the price increase of a single product in the candidate market and that do not require aggregating differentiated products.[35] The asymmetric price increase approach is a relatively

[35] Although Katz and Shapiro state that their formulation applies when the price of only one product is increased, Øystein Daljord, Lars Sørgard, and Øyvind Thomassen correctly point out that the result in Katz and Shapiro applies to a symmetric price increase among all products, but not to an asymmetric price increase. See Michael L. Katz and Carl Shapiro, "Critical Loss: Let's Tell the Whole Story," *Antitrust Magazine*, ABA Section of the Antitrust Law, Spring 2003, pages 49-56 at page 53 and Øystein Daljord, Lars Sørgard & Øyvind Thomassen, "The SSNIP Test and

simple extension of the standard critical loss framework that only requires one additional piece of information, the diversion ratio between the two relevant products. For example, the asymmetric critical loss for product 1 would be

$$\mathrm{CL}_1 = \frac{t_1}{t_1 + m_1 - m_2 \frac{p_2}{p_1} D_{12}} \tag{24}$$

where D_{12} is the diversion ratio between product 1 and 2.[36] Since this additional input is frequently estimated because it is required for other common antitrust analyses like the Gross Upward Pricing Pressure Index ("GUPPI"), the critical loss analysis with asymmetric price increases proposed by Langenfeld and Li and Daljord et al. is a practical approach for conducting the hypothetical monopolist test.[37]

Another alternative is to calculate the GUPPI. The GUPPI for product 1 is defined as

$$GUPPI_1 = D_{12} \frac{p_2 - c_2}{p_1} = D_{12} m_2 \frac{p_2}{p_1} \tag{25}$$

where D_{12} is again the diversion ratio of the unit sales of product 1 to product 2 when there is a price increase for product 1.[38] The GUPPI for product 2 is defined symmetrically. The higher the GUPPI, the larger the price increase a hypothetical monopolist could profitably impose.[39] However, the GUPPI itself

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Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro," *Journal of Competition Law & Economics*, Vol. 4, 2008, pages 263-270.

[36]See James Langenfeld and Wenqing Li, "Critical Loss Analysis in Evaluating Mergers," *The Antitrust Bulletin*, Summer 2001, pages 299-337 at page 316.

[37] For details on how to conduct critical loss analysis with asymmetric price increase, see James Langenfeld and Wenqing Li, "Critical Loss Analysis in Evaluating Mergers," *The Antitrust Bulletin*, Summer 2001, pages 299-337; Øystein Daljord, Lars Sørgard, and Øyvind Thomassen, "The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro," *Journal of Competition Law & Economics*, Vol. 4, 2008, pages 263-270; James Langenfeld and Wenqing Li, "Asymmetric Price Increase in Critical Loss Analysis: A Reply to Daljord, Sørgard, and Thomassen," *Journal of Competition Law & Economics*, 10(2), 2014, pages 495-503; and James Langenfeld and Wenqing Li, "Asymmetric Price Increase in Critical Loss Analysis: Surreply to Daljord, Sørgard, and Thomassen," *Journal of Competition Law & Economics*, 10(3), 2014, pages 769-772.

[38] For a discussion of GUPPI, see Serge Moresi, "The Use of Upward Price Pressure Index in Merger Analysis", The Antitrust Source, February 2010, pages 1-12 at pages 6-7.

[39] The rate at which the GUPPI is passed through into a price increase is held constant as we discuss below.

is not a predicted percentage increase in price and to translate the GUPPI into a percentage increase in price, one needs to make an assumption about the rate at which the GUPPI passes through into a price increase. Using Monte Carlo simulations, recent work by Miller et al. (2017) shows how the pass-through rate of the GUPPI into a price increase varies with the demand system and suggests that a pass through rate of one is reasonable for many demand systems.[40] A pass through rate of one implies that a one percentage point increase in the GUPPI would result in a one percent increase in price. This means that we can evaluate whether the GUPPI for product 1 or product 2 is greater than or equal to 5% to assess if an asymmetric price increase of at least five percent is likely to be profitable.

Tables 3 and 4 compare the simulation results for the differentiated products critical loss analysis of the profitability of a common price increase versus the results for the critical loss analysis with asymmetric price increases and the GUPPI. As shown in Table 3, when the differentiated products critical loss analysis indicates a 5% common price increase is profitable, critical loss analysis with asymmetric price increases also shows that a 5% price increase is profitable for either product 1 or product 2 except for some rare occasions. Similarly, in these instances, the GUPPI for product 1 or product 2 typically exceeds 5%, indicating a hypothetical monopolist would be likely to raise the price of at least one of the products post-merger by at least 5% (see Table 4).

However, when the differentiated products critical loss analysis indicates a 5% common price increase is not profitable, the GUPPI for product 1 or product 2 is often greater than or equal to 5%, signaling upward price pressure. Consistent with the GUPPI, the critical loss analysis with asymmetric price increase also indicates that it can be profitable to impose a 5% price increase for product 1 only or

[40] Nathan Miller, Marc Remer, Conor Ryan, and Gloria Sheu. 2017. "Upward Pricing Pressure as a Predictor of Merger Price Effects*," International Journal of Industrial Organization*, (52):216-247. Similarly, some economists have suggested that using 5% or 10% GUPPI as a threshold to evaluate a merged party is likely to impose a price increase. See Steven C. Salop, Serge X. Moresi, and John R. Woodbury, "Scoring Unilateral Effects with the GUPPI: The Approach of the New Horizontal Merger Guidelines," CRA Competition Memo, Charles River Associates, August 31, 2010, at page 2.

for product 2 only even though a 5% common price is not profitable. The results in Tables 3 and 4 show that the common price increase approach produces markets that are weakly broader than the asymmetric price increase approach. Since the Merger Guidelines only require the hypothetical monopolist to be able to impose a profitable price increase on at least one product in the market,[41] the asymmetric price increase critical loss and the GUPPI approach to defining a market are consistent with a strict application of the Merger Guidelines, while the common price increase approach imposes a higher standard for defining a market.

These results indicate that the standard critical loss formula result in markets that are too broad when products are differentiated for two reasons. First, as we showed in the previous section, it sets the bar too high relative to an approach that allows for product differentiation. Second, it imposes a common price increase, which is a higher bar than required by the Merger Guidelines.[42]

The consistency between results based on the GUPPI and those based on critical loss with asymmetric price increase is not surprising. As can be seen in Equations (24) and (25), the GUPPI is the third term in the denominator in the formula for critical loss with asymmetric price increases, which is subtracted from the first two terms in the denominator. When the GUPPI becomes larger, the denominator for critical loss with asymmetric price increases becomes smaller, which in turn would lead to a larger value for the critical loss with asymmetric price increases. In other words, the larger the GUPPI, the larger the critical loss with asymmetric price increases, both of which suggest an asymmetric price increase is more likely to be profitable. As shown in Table 5, when the GUPPI for both product 1 and

[41] Horizontal Merger Guidelines, U.S. Department of Justice and the Federal Trade Commission, August 19, 2010, section 4.1.1.

[42] In certain instances where one can identify the smallest possible candidate antitrust market in advance, the standard critical loss analysis under a representative product approach can be useful as a first pass. If the standard critical loss analysis indicates the candidate market is an antitrust market, then alternative analyses would likely not be necessary since both critical loss analysis with asymmetric price increase and GUPPI would mostly likely provide the same answer and the practitioner knows that it is not practical to define an antitrust market that is even smaller.

product 2 are lower than 5%, the critical loss with asymmetric price increase also indicates a 5% price increase is not profitable for product 1 or product 2 with rare exceptions. On the other hand, when GUPPI for either product 1 or product 2 equal to or exceeds 5%, it is almost always profitable to raise price for product 1 or product 2 by 5% according to the critical loss with asymmetric price increase.

Appendix

1. **Proof that the standard critical loss analysis and the differentiated products critical loss analysis of the profitability of a common price increase are equivalent under product symmetry.**

Under product symmetry, we have $p_1 = p_2 = p$, $C_1 = C_2 = C$, and $\Delta q_1 = \Delta q_2 = \Delta q$. Then (4) becomes:

$$2tpq \geq 2p(1+t)\Delta q - 2C\Delta q \tag{A1}$$

Divide both sides of (A1) by $2q$, we have:

$$tp \geq [p(1+t) - C]\frac{\Delta q}{q} \tag{A2}$$

Divide both sides of (A2) by $[p(1+t) - C]$, we have:

$$\frac{tp}{p(1+t)-C} \geq \frac{\Delta q}{q} \tag{A3}$$

Divide the right side of (A3) by p and use the fact that $\frac{\Delta q}{q} = \frac{2\Delta q}{2q} = \frac{\Delta Q}{Q}$, we have (A4) below which is the same as (2).

$$\frac{t}{t+m} \geq \frac{\Delta Q}{Q} \tag{A4}$$

Please note that the percentage change in quantity when the prices of both products increase are different from the percentage change in quantity when the price of one product rises only.

2. Derivation of the inequality for the differentiated products critical loss analysis of profitability of a common price increase

From (3), we have:

$$p_1(1+t)q_1 - p_1(1+t)\Delta q_1 + C_1\Delta q_1 + p_2(1+t)q_2 - p_2(1+t)\Delta q_2 + C_2\Delta q_2$$

$$\geq p_1q_1 + p_2q_2 \qquad \text{(A5)}$$

From (A5), we have

$$tp_1q_1 - p_1(1+t)\Delta q_1 + C_1\Delta q_1 + tp_2q_2 - p_2(1+t)\Delta q_2 + C_2\Delta q_2 \geq 0 \qquad \text{(A6)}$$

Move some terms to the right side of inequality (A6), we have (4).

The profit maximization for product 1 before the common price increase implies the Lerner Index:

$$\frac{p_1 - C_1}{p_1} = \frac{1}{\varepsilon_1} \qquad \text{(A6)}$$

solve for $p_1$using (A6), we have (5). (6) can be derived similarly.

Substitute (5)-(8) into the right of inequality (4), the right side of inequality (4) becomes:

$$(1+t)\left[\frac{C_1\varepsilon_1}{\varepsilon_1-1}(q_1t\varepsilon_1 - q_1t\varepsilon_{12}) + \frac{C_2\varepsilon_2}{\varepsilon_2-1}(q_2t\varepsilon_2 - q_2t\varepsilon_{21})\right]$$

$$-[C_1(q_1t\varepsilon_1 - q_1t\varepsilon_{12}) + C_2(q_2t\varepsilon_2 - q_2t\varepsilon_{21})] \qquad \text{(A7)}$$

Re-arrange the terms in (A7), (A7) becomes:

$$\left[(1+t)\frac{\varepsilon_1}{\varepsilon_1-1} - 1\right][C_1(q_1t\varepsilon_1 - q_1t\varepsilon_{12})] + \left[(1+t)\frac{\varepsilon_2}{\varepsilon_2-1} - 1\right][C_2(q_2t\varepsilon_2 - q_2t\varepsilon_{21})] \qquad \text{(A8)}$$

Re-arrange the terms in (A8), (A8) becomes:

$$\frac{t\varepsilon_1+1}{\varepsilon_1-1}[C_1(q_1t\varepsilon_1-q_1t\varepsilon_{12})]+\frac{t\varepsilon_2+1}{\varepsilon_2-1}[C_2(q_2t\varepsilon_2-q_2t\varepsilon_{21})] \tag{A9}$$

Substitute (A9) as the right side of inequality (4) and substitute (5) and (6) to the left side of inequality (4), we have inequality (9).

3. Derivation of the price-cost margin of the single aggregated product

Substitute (11) and (12) into (13), we have:

$$\overline{m}=\frac{\frac{p_1q_1+p_2q_2}{q_1+q_2}-\frac{C_1q_1+C_2q_2}{q_1+q_2}}{\frac{p_1q_1+p_2q_2}{q_1+q_2}}=\frac{(p_1-C_1)q_1+(p_2-C_2)q_2}{p_1q_1+p_2q_2} \tag{A10}$$

Substitute (5) and (6) into (A10), we have:

$$\overline{m}=\frac{\left(\frac{C_1\varepsilon_1}{\varepsilon_1-1}-C_1\right)q_1+\left(\frac{C_2\varepsilon_2}{\varepsilon_2-1}-C_2\right)q_2}{\frac{C_1\varepsilon_1}{\varepsilon_1-1}q_1+\frac{C_2\varepsilon_2}{\varepsilon_2-1}q_2}=\frac{C_1q_1\left(\frac{\varepsilon_1}{\varepsilon_1-1}-1\right)+C_2q_2\left(\frac{\varepsilon_2}{\varepsilon_2-1}-1\right)}{C_1q_1\frac{\varepsilon_1}{\varepsilon_1-1}+C_2q_2\frac{\varepsilon_2}{\varepsilon_2-1}}=\frac{C_1q_1\frac{1}{\varepsilon_1-1}+C_2q_2\frac{1}{\varepsilon_2-1}}{C_1q_1\frac{\varepsilon_1}{\varepsilon_1-1}+C_2q_2\frac{\varepsilon_2}{\varepsilon_2-1}} \tag{A11}$$

4. Derivation of standard critical loss

A price increase for a representative product will be profitable whenever

$$p(1+t)(q-\Delta q)-C(q-\Delta q)\geq pq-Cq \tag{A12}$$

where $p=\bar{p}$ and $C=\bar{C}$ as $\bar{p}$ and $\bar{C}$ are defined in (11) and (12) and $\Delta q=\Delta q_1+\Delta q_2$.

Following equation (4), this simplifies to.

$$tpq\geq p(1+t)\Delta q-C\Delta q=[p(1+t)-C]\Delta q \tag{A13}$$

We can then divide by $[p(1+t)-C]$ to get

$$\frac{tp}{[p(1+t)-C]}\geq\frac{\Delta q}{q} \tag{A14}$$

With some additional algebra we obtain

$$\frac{t}{\frac{p-c}{p}+t} \geq \frac{\Delta q}{q} \tag{A15}$$

Which is just the standard critical loss formula defined in equation (2) where $m = \frac{p-c}{p}$

$$\frac{t}{m+t} \geq \frac{\Delta q}{q} \tag{A16}$$

5. Necessary condition for when differentiated product critical loss differs from standard critical loss

Inequality (4) shows a common price increase of t percent is only profitable if the following condition holds.

$$tp_1q_1 + tp_2q_2 \geq [p_1(1+t) - C_1]\Delta q_1 + [p_2(1+t)\Delta - C_2]\Delta q_2 \tag{A17}$$

The standard critical loss analysis will only define the same market as the differentiated product critical loss analysis when this inequality is identical to the inequality for standard critical loss. To see when they are the same, start with the equivalent inequality for a representative product from (A13), is $tpq \geq p(1+t)\Delta q - C\Delta q$ and rewrite it assuming you define the representative product as an average. Then you have

$$t\frac{p_1q_1+p_2q_2}{q_1+q_2}(q_1+q_2) \geq \frac{p_1q_1+p_2q_2}{q_1+q_2}(1+t)(\Delta q_1+\Delta q_2) - \frac{C_1q_1+C_2q_2}{q_1+q_2}(\Delta q_1+\Delta q_2) \tag{A18}$$

Which simplifies to

$$tp_1q_1 + tp_2q_2 \geq \left[\frac{p_1q_1+p_2q_2}{q_1+q_2}(1+t) - \frac{C_1q_1+C_2q_2}{q_1+q_2}\right](\Delta q_1+\Delta q_2) \tag{A19}$$

$$tp_1q_1 + tp_2q_2 \geq \left[\frac{[p_1(1+t)-C_1]q_1}{q_1+q_2} + \frac{[p_2(1+t)-C_2]q_2}{q_1+q_2}\right](\Delta q_1+\Delta q_2) \tag{A20}$$

$$tp_1q_1+tp_2q_2 \geq [p_1(1+t)-C_1]\frac{q_1}{q_1+q_2}(\Delta q_1+\Delta q_2)+[p_2(1+t)-C_2]\frac{q_2}{q_1+q_2}(\Delta q_1+\Delta q_2) \tag{A21}$$

Notice that the left sides of inequality (A17) and (A21) are identical and the right side of inequality (A22) is similar to (A17), but has different weights on each of the profit terms. From this we can infer that in order for differentiated product critical loss to differ from standard critical loss, it must be the case that the two products have different per unit profits. I.e., it must be the case that

$$p_1(1+t) - C_1 \neq p_2(1+t) - C_2 \tag{A22}$$

6. How Differentiated Product Critical Loss Differs from Standard Critical Loss When Per Unit Profits are Different

Consider the case where the two products have different per unit profits, i.e., that $p_1(1+t) - C_1 \neq p_2(1+t) - C_2$. By comparing equations (A17) and (A21) we can see that the weight that standard critical loss places on product 1 differs from differentiated product critical loss whenever

$$\frac{q_1}{q_1+q_2}(\Delta q_1 + \Delta q_2) \neq \Delta q_1 \tag{A23}$$

Which can be simplified to

$$\frac{q_1}{q_1+q_2}(\Delta q_1 + \Delta q_2) - \Delta q_1 \neq 0 \tag{A24}$$

$$\frac{q_1(\Delta q_1+\Delta q_2)}{q_1+q_2} - \frac{\Delta q_1(q_1+q_2)}{q_1+q_2} \neq 0 \tag{A25}$$

$$\frac{q_1\Delta q_2 - \Delta q_1 q_2}{q_1+q_2} \neq 0 \tag{A26}$$

Using equations (7) and (8) we can re-write this as

$$\frac{q_1(q_2 t\varepsilon_2 - q_2 t\varepsilon_{21}) - (q_1 t\varepsilon_1 - q_1 t\varepsilon_{12})q_2}{q_1+q_2} \neq 0 \tag{A27}$$

$$\frac{q_1 q_2 t(\varepsilon_2 - \varepsilon_{21}) - q_1 q_2 t(\varepsilon_1 - \varepsilon_{12})}{q_1+q_2} \neq 0 \tag{A28}$$

$$\frac{q_1 q_2 t[(\varepsilon_2 - \varepsilon_1) - (\varepsilon_{21} - \varepsilon_{12})]}{q_1+q_2} \neq 0 \tag{A29}$$

7. Lower Own Price Elasticity Implies Higher Price Holding Cost Constant

From (5) and (6), we have

$$p_1 - p_2 = \frac{C_1\varepsilon_1}{\varepsilon_1 - 1} - \frac{C_2\varepsilon_2}{\varepsilon_2 - 1} \tag{A30}$$

If we assume costs are the same, we have

$$p_1 - p_2 = C\left[\frac{\varepsilon_1(\varepsilon_2 - 1)}{(\varepsilon_1 - 1)(\varepsilon_2 - 1)} - \frac{\varepsilon_2(\varepsilon_1 - 1)}{(\varepsilon_2 - 1)(\varepsilon_1 - 1)}\right] = C\left[\frac{\varepsilon_2 - \varepsilon_1}{(\varepsilon_1 - 1)(\varepsilon_2 - 1)}\right] \tag{A31}$$

Therefore, if $\varepsilon_1 < \varepsilon_2$, we have $p_1 > p_2$.

8. Lower Own Price Elasticity Implies Lower Cost Holding Price Constant

From (5) and (6), we have

$$C_1 - C_2 = \frac{p_{1(}\varepsilon_1 - 1)}{\varepsilon_1} - \frac{P_2(\varepsilon_2 - 1)}{\varepsilon_2} \tag{A32}$$

If we assume prices are the same, we have

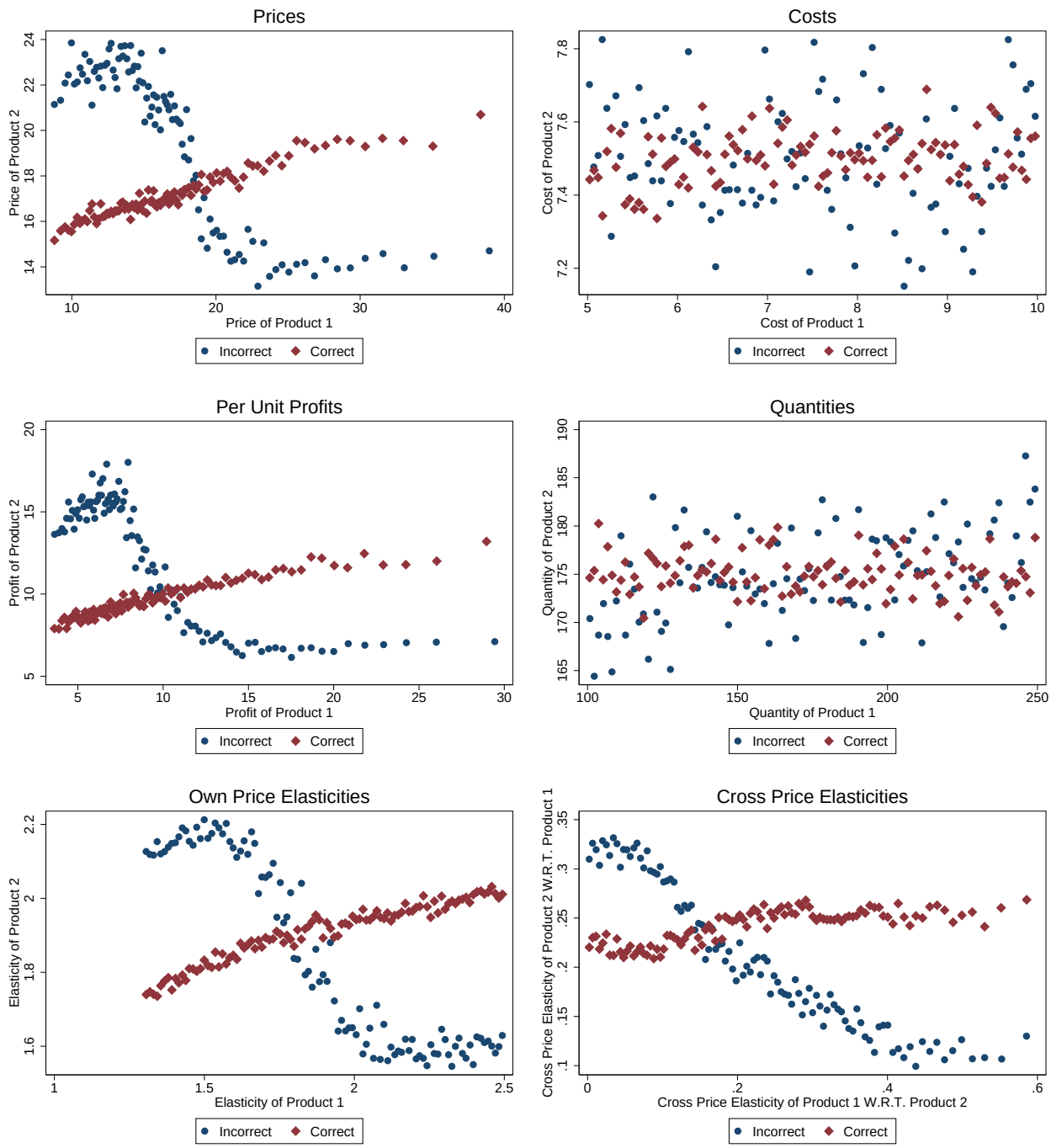
$$C_1 - C_2 = p\left[\frac{\varepsilon_2(\varepsilon_1 - 1)}{\varepsilon_1\varepsilon_2} - \frac{\varepsilon_1(\varepsilon_2 - 1)}{\varepsilon_1\varepsilon_2}\right] = p\left[\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1\varepsilon_2}\right] \tag{A33}$$

Therefore, if $\varepsilon_1 < \varepsilon_2$, we have $C_1 < C_2$.

Table 1: Likelihood of Divergence between Standard Critical Loss Analysis and Differentiated Products Critical Loss Analysis

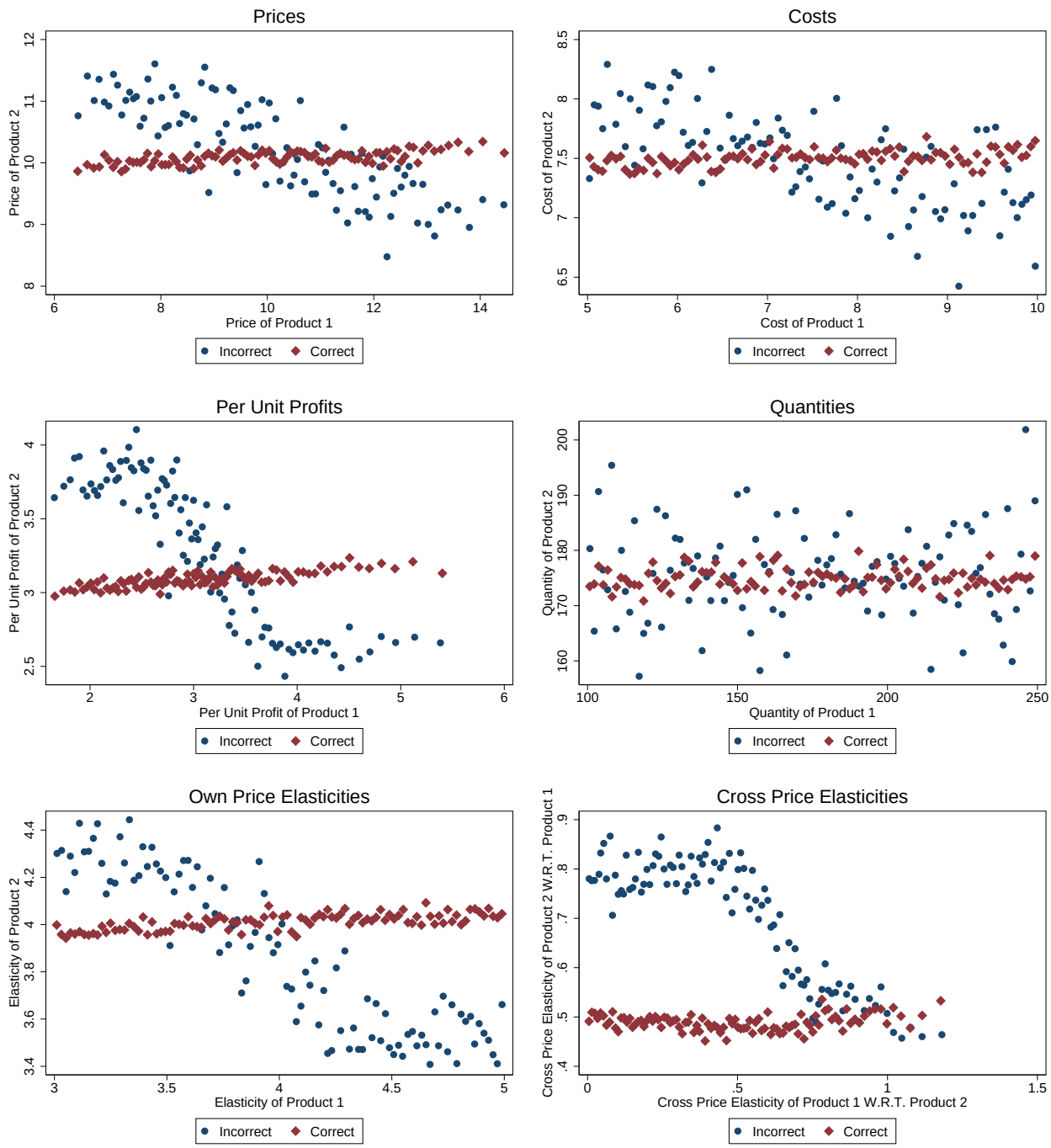
Is a 5% Price Increase Profitable with the Standard Critical Loss Analysis?	**Is a 5% Price Increase Profitable with the Differentiated Products Critical Loss Analysis?**	**Number of Instances**	**Percentage of Instances**
Profitable	Profitable	27,886	55.77%
Not Profitable	Not Profitable	12,428	24.86%
Profitable	Not Profitable	344	0.69%
Not Profitable	Profitable	9,342	18.68%

Figure 1: Binned Scatter Plots of Firm Characteristics in Simulations where Standard Critical Loss Makes Correct or Incorrect Predictions (Own Price Elasticities from 1.3-2.5)



Notes: Binned scattered plots of simulated firm characteristics for cases where critical loss correctly or incorrectly predicts whether a price increase will be profitable.

Figure 2: Binned Scatter Plots of Firm Characteristics in Simulations where Standard Critical Loss Makes Correct or Incorrect Predictions (Own Price Elasticities from 3-5)



Notes: Binned scattered plots of simulated firm characteristics for cases where critical loss correctly or incorrectly predicts whether a price increase will be profitable.

Table 2: Likelihood of Divergence between Standard Critical Loss Analysis and Differentiated Products Critical Loss Analysis with Different Own Elasticity Ranges

Is a 5% Price Increase Profitable with the Standard Critical Loss Analysis?	**Is a 5% Price Increase Profitable with the Differentiated Products Critical Loss Analysis?**	**Own Elasticity: 1 to 2**	**Own Elasticity: 1.2 to 2.5 (Base Case)**	**Own Elasticity: 1.5 to 3**	**Own Elasticity: 3 to 5**
		Percentage of Instances	**Percentage of Instances**	**Percentage of Instances**	**Percentage of Instances**
Profitable	Profitable	55.16%	55.77%	48.32%	18.89%
Not Profitable	Not Profitable	17.13%	24.86%	33.36%	74.93%
Profitable	Not Profitable	0.44%	0.69%	0.77%	0.78%
Not Profitable	Profitable	27.27%	18.68%	17.55%	5.41%

Table 3: Comparison of Differentiated Products Critical Loss Analysis and Critical Loss with Asymmetric Price Increase

Is a 5% Common Price Increase Profitable with the Differentiated Products Critical Loss Analysis?	**Is a 5% Prince Increase Profitable for Product 1 or Product 2 Based on Critical Loss Analysis with Asymmetric Price Increase?**	**Number of Instances**	**Percentage of Instances**
Profitable	Yes	37,062	74.12%
Profitable	No	166	0.33%
Not Profitable	Yes	5,548	11.10%
Not Profitable	No	7,224	14.45%

Table 4: Comparison of Differentiated Products Critical Loss Analysis and GUPPI

Is a 5% Common Price Increase Profitable with the Differentiated Products Critical Loss Analysis?	Is GUPPI for Product 1 or Product 2 Greater Than or Equal to 5%?	Number of Instances	Percentage of Instances
Profitable	Yes	37,114	74.23%
Profitable	No	114	0.23%
Not Profitable	Yes	5,548	11.10%
Not Profitable	No	7,224	14.45%

Table 5: Comparison of GUPPI and Critical Loss with Asymmetric Price Increase

Is GUPPI for Product 1 or Product 2 Greater Than or Equal to 5%?	Is a 5% Prince Increase Profitable for Product 1 or Product 2 Based on Critical Loss Analysis with Asymmetric Price Increase?	Number of Instances	Percentage of Instances
Yes	Yes	42,610	85.22%
Yes	No	52	0.10%
No	Yes	0	0.00%
No	No	7,338	14.68%